Intermediate Micro In-Class Problems

Monopoly I

June 21, 2016

Yangrou Paomonopoly

The monopoly supplier of Rangyou Paomo in Xian has production costs of a constant ¥2 per unit. For a unit price of p, demand for Rangyou Paomo will be $100 - p^2$.

1. Write down the monopolist's profit as a function of p.
2. Compute the profit-maximizing price she should charge.
3. What is the elasticity of demand at the profit maximizing price? (*Note: if demand is given by $D(p)$, the elasticity of demand is given by the following:*)

$$\varepsilon(p) = -\frac{\partial D}{\partial p}\frac{p}{D(p)}$$

1. Profit is given by:

$$\Pi(p) = (p-2)\left(100 - p^2\right)$$

(i.e., markup times quantity)

2. The first order condition on maximizing profits by choosing p is:

$$100 - 3p^2 + 4p = 0 \Rightarrow p^* = \frac{4+\sqrt{1216}}{6} \approx 6.48$$

3. We know p in the formula; we need an expression for $\frac{\partial D}{\partial p}$ and to find $D(p)$.

$$D(p) = 100 - (p^*)^2 = \frac{592 - 4\sqrt{304}}{9} \approx 58.03$$
$$\frac{\partial D}{\partial p} = -2p$$

So we have

$$\varepsilon(p) = \frac{2p^2}{100 - p^2}$$

$$\varepsilon(p^*) = \frac{154 + 2\sqrt{304}}{148 - \sqrt{304}} \approx 1.45$$

More Monopoly

Consider a monopolist with production cost function $C(q) = 640 + 20q$, where q is the quantity produced. Let $D(p) = 50 - \frac{p}{2}$ be the demand-price relationship.

1. What is the elasticity of demand at the price $p = 20$?
2. At the price $p = 44$, if the monopolist wishes to raise revenue, should he raise or lower the price?
3. What is the monopolist's maximum profit?
4. What is the elasticity of demand at the profit-maximizing price?

1. Using the formula from above:

$$\varepsilon(20) = -\left(-\frac{1}{2}\right)\frac{20}{40} = \frac{1}{4}$$

2. At the optimum, the derivative of profit with respect to price is 0; since this function is concave, the direction of the derivative will tell us how to adjust our price.

$$\Pi(p) = (50 - \frac{p}{2})p - 640 - 20(50 - \frac{p}{2}) = (p - 20)(50 - \frac{p}{2}) - 640$$

So that

$$\Pi'(p) = 60 - p;$$

$$\Pi'(20) = 40 > 0$$

So we should raise prices.

3. We quickly see form above that $p^* = 60$, from which we quickly deduce:

$$\Pi(p^*) = \frac{1}{2}40^2 - 640 = 160$$

4. Plugging into the above:

$$\varepsilon(60) = -\left(-\frac{1}{2}\right)\frac{60}{20} = \frac{3}{2}$$

Choosing Price vs. Choosing Quantity

Consider a monopolist facing a demand curve of the form $D = 50 - 3p$ where p is the unit price. Suppose the monopolist has a constant marginal cost of production of ¥3 per unit.

1. Instead of choosing a unit price p to maximize profit, our monopolist will choose a quantity q to maximize profit. Write down, as function of q, the price per unit the monopolist must charge to sell exactly q units. This object is called the **inverse demand curve**.
2. Write down, as a function of q, the monopolist's profit.
3. Write down, as a function of q, the monopolist's marginal revenue.
4. Use either the function you identified in part (2) or in part (3) to compute the profit-maximizing quantity. What is it?
5. For your own edification, check that you reach the same conclusion by choosing a profit-maximizing price insted.

1.
$$p = \frac{50 - q}{3}$$

2.
$$\Pi(q) = \frac{50 - q}{3}q - 3q$$

3.
$$MR(q) = \frac{\partial R(q)}{\partial q} = \frac{50 - 2q}{3}$$

4.
$$\Pi(q) = \frac{1}{3}(41 - q)q \Rightarrow q^* = \frac{41}{2}$$

5.
$$\Pi(p) = (p - 3)(50 - 3p) \Rightarrow p^* = \frac{59}{6} \Rightarrow q^* = 50 - \frac{59}{2} = \frac{41}{2}$$

Returns to Scale

Let $C(q)$ denote the total cost incurred to produce q units of Tsingtao. Decide which of the following cost functions exhibit constant, decreasing and increasing returns to scale.

1. $C(q) = 5q + 3$ for $q \geq 0$.
2. $C(q) = 2q^2 + 3q + 1$.
3. $C(q) = 5q - q^2$ for $q \leq 5$.
4. $C(q) = 5q^{\frac{1}{2}}$

Returns to scale are determined by the sign of the second derivative of the cost function. If $C''(q) < 0$, returns to scale are increasing; if it's positive, the returns are decreasing; and if they're zero, returns are constant.

1. $C''(q) = 0 \Rightarrow CRS$
2. $C''(q) = 4 > 0 \Rightarrow DRS$
3. $C''(q) = -2 < 0 \Rightarrow IRS$
4. $C''(q) = -\frac{5}{4}q^{-\frac{3}{2}} < 0 \Rightarrow IRS$